Foxyards Primary School

+ − Written Calculation Policy x ÷

 C. Weaver, L. Hughes & L. Evans – January 2016

Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation for addition** which they know they can rely on **when mental methods are not appropriate.**

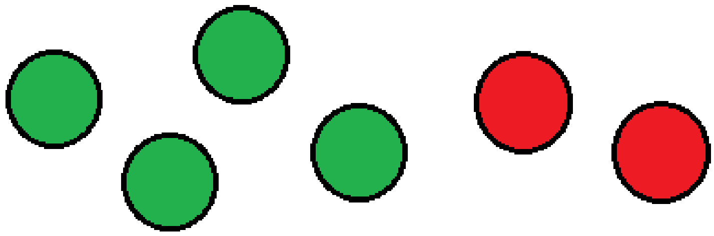
To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

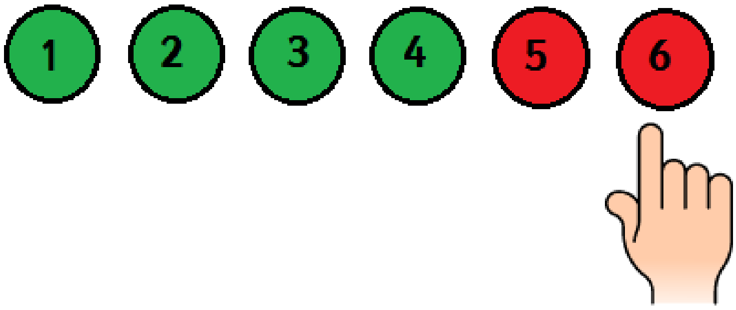
Stage 1: Using objects and pictures. (AP3 onwards).

Aggregation

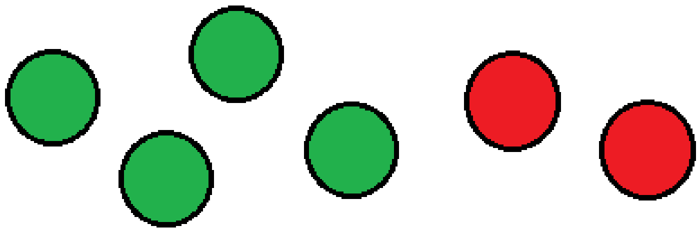


If I have 4 green counters and 2 red, how many altogether?

Combine two sets of objects and count them all, pointing with a finger to ensure none are missed out.

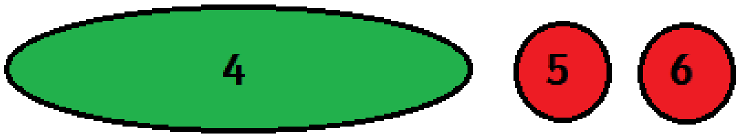


Augmentation



If I have 4 green counters and then add 2 more, how many in total?

Add to a set by counting on from what you already know is in the first set.



Stage 2: Early practical addition (AP7 onwards)

Recording practical methods (using cubes or counters) as number sentences.

8 + 7 = 15
7 + 6 = 13

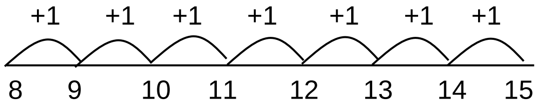
Put the largest number in your head, then use objects or your fingers to count on.

Stage 3: Addition on a number line (AP10 onwards)

Steps in addition can then be recorded on a number line. The steps often bridge through a multiple of 10

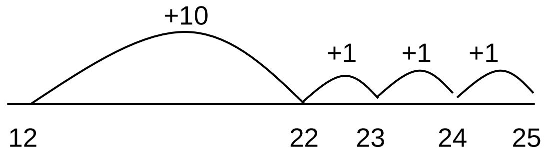
Begin by counting on in 1s on a number line.

8 + 7 = 15

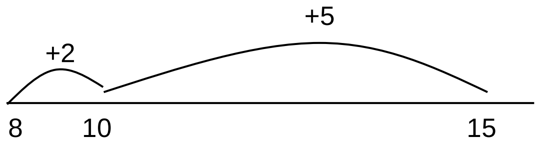


This is then extended to adding groups of 10, on a blank number line.

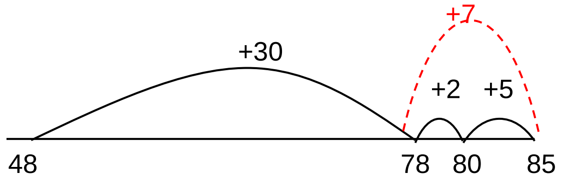
12 + 13 = 25



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.
8 + 7 = 15



48 + 37 = 85



Partitioning (optional mental method)

Record steps in addition using partitioning:
Initially as a jotting: -

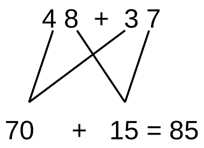
48 + 37 =
40 + 30 = 70
8 + 7 = 15
70 + 15 = 85

This method is basically a 'jotting' version of the number line method

Or

48 + 37 =
40 + 30 + 8 + 7 =
70 + 15 = 85

One popular jotting approach is: -



Stage 4: Column method (AP17 onwards)

58 + 87 **457 + 76** **538 + 286**

```
  58    Then    457   Then    538
+ 87          +  76         + 286
 145            533           824
 11             11            11
```

Record carry digits below the line. Use the words 'carry ten' and 'carry hundred', not 'carry one'

Once confident, use with larger whole numbers and decimals (AP 21).

2467 + 785 **4824 + 2369** **46.73 + 78.6**

```
   2467        4824        46.73
 +  785     +  2369     +  78.60
   3252        7193       125.33
   111         1  1       111
```

When adding decimals, the decimal points should align, just like the hundreds, tens and ones.

By Year 6, children should be able to add 6-digit whole numbers and decimals up to 3 decimal places.

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for** subtraction which they know they can rely on **when mental methods are not appropriate.**

But, they should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 2006 -128 then the 'counting on' approach may well be the best method in that particular instance

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies: -

Taking away (Counting Back) ***Complementary Addition (Counting On)***

When should we count back and when should we count on?

This will alter depending on the calculation (see below), but often the following rules apply

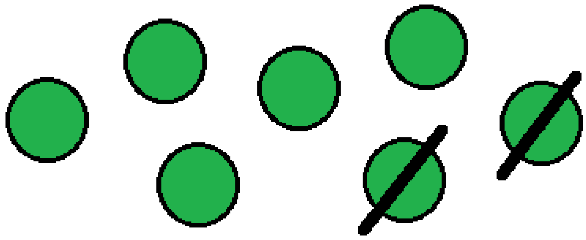
If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

If the numbers are close together (206 – 188), then count up

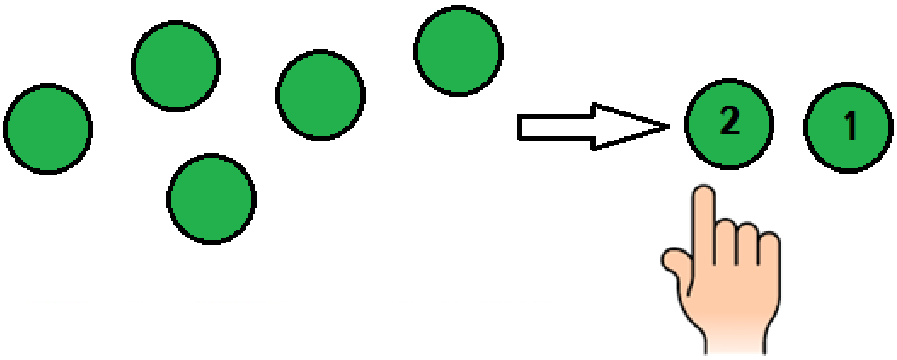
In many cases, either strategy would be suitable

Stage 1: Using objects and pictures (AP3 onwards)

Removing items from a set using Take away or Reduction



Count out the first number of counters and then remove or take away the second number to find out how many are left. E.g. 7 – 2 = 5



Stage 2: Early practical subtraction (AP9 onwards)

Recording practical methods (using cubes or counters) as number sentences.

12 - 7 = 5
17 - 9 = 8

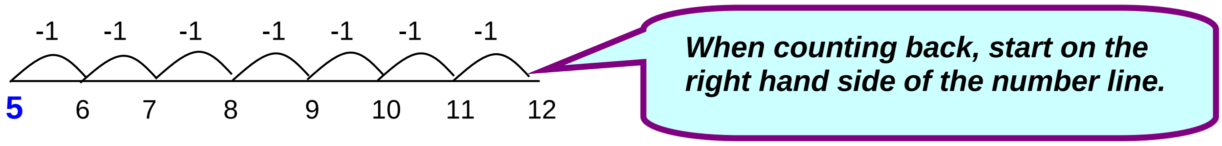
Use objects or your fingers to count back.

Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
Stage 3: Subtraction on a number line (AP10 onwards)	

The number line helps to record or explain the steps in mental subtraction.
It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly.
It can also show **counting up** from the smaller to the larger number to **find the difference,**

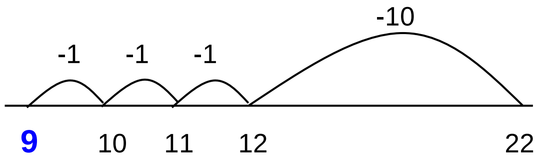
Begin by **counting back** in 1s on a number line.

12 - 7 = 5



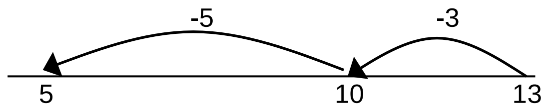
This is then extended to subtracting groups of 10, on a blank number line.

22 - 13 = 9



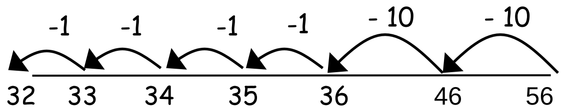
The steps often bridge through a multiple of 10.

13 – 8 = 5

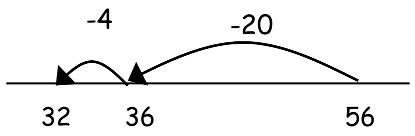


For 2 digit numbers, count back in 10s and 1s

56 – 24 = 32

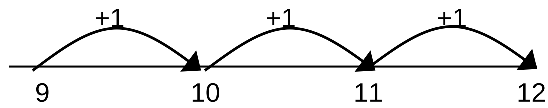


Then subtract tens and units in single jumps



Small differences can be found by counting up

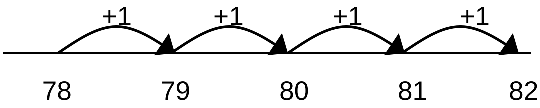
12 – 9 = 3



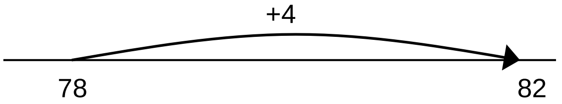
For 2 (or 3) digit numbers close together, count up

82 – 78 = 4

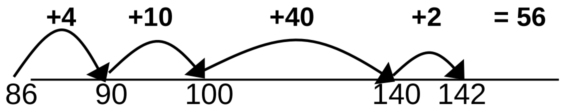
First, count in ones



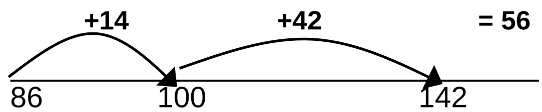
Then, use number facts to count in a single jump



142 – 86



Or (in fewer steps)



Stage 4: Column method (AP18 onwards)

84 – 46

```
  7   1
  8   4
- 4   6
  3   8
```

854 – 286

```
  7  14  1
  8   5  4
- 2   8  6
  5   6  8
```

Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. Eight hundred and fifty-four subtract two hundred and eighty-six.

Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)

605 – 328

```
      9
  5   1   1
  6   0   5
- 3   2   8
  2   7   7
```

Move onto examples using 4 digit (or larger) numbers and then onto decimal calculations (from AP21), such as money or length.

8146 – 4729

```
  7   1  3   1
  8   1  4   6
- 4   7  2   9
  3   4  1   7
```

£8.46 – £7.29

```
       3   1
  £8 . 4   6
- £7 . 2   9
  £1 . 1   7
```

13.4cm – 8.7cm

```
  0  12   1
  1   3 . 4 cm
-     8 . 7 cm
  0   4 . 7 cm
```

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 10 × 10;**
- **partition number into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note:

Children need to acquire **one efficient written method of calculation for** multiplication which they know they can rely on **when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

These mental methods are often more efficient than written methods when multiplying.

Use partitioning and grid methods until number facts and place value are secure

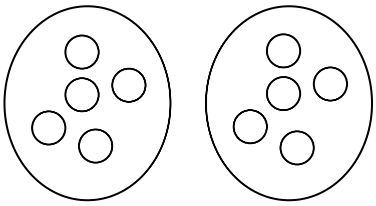
For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

When multiplying a 2 digit x 3 digit number (or a 3digit x 3 digit number), the standard method is usually the most efficient

At all stages, use known facts to find other facts. E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16)

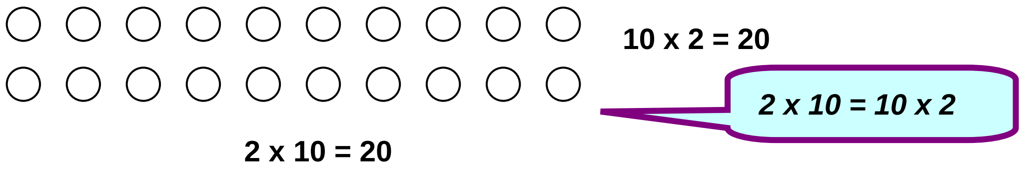
Stage 1: Practical mental methods (AP10 onwards)

Begin by using practical equipment (cubes or counters) to introduce the ‘x’ symbol through making groups, e.g. 2 x 5 = 2 groups of 5.



2 x 5 = 10

Using an array



Practise counting in multiples of 2, 5 and 10.

0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20…

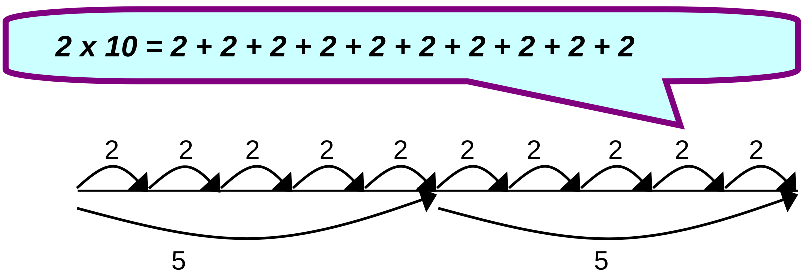
0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50...

0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100…

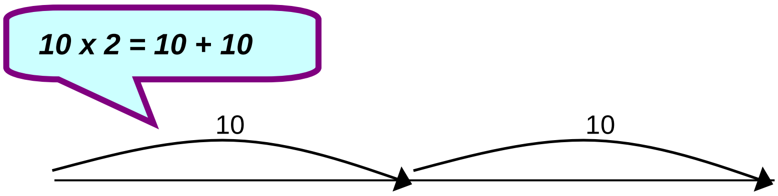
Stage 2: Repeated addition on a number line (AP12 onwards)

Begin by building on the understanding that multiplication is repeated addition, using arrays and number lines to support the thinking.

Using a number line



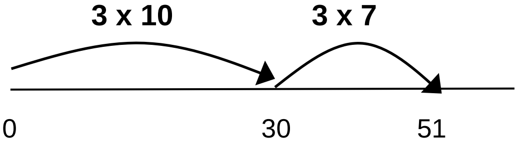
Or



Extend the above methods to include the 3, 4 and 6 times tables then begin to partition using **jottings and number lines.**

17 x 3

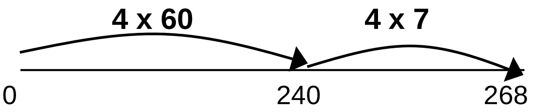
10 x 3 = 30
7 x 3 = <u>21</u>
51



Extend the methods above to calculations which give products greater than 100.

4 x 67

60 x 4 = 240
7 x 4 = 28
268



Each of these methods can be used in the future if children find expanded or standard methods difficult

Extend to using these methods with all tables to 10 x 10.

Stage 3: Short multiplication method (AP18 onwards)

The expanded method links the grid method to the standard method.
It still relies on partitioning the tens and units, but sets out the products vertically.

Children will use the expanded method until they can securely use and explain the standard method.

When setting out calculations vertically, begin with the ones first (as with addition and subtraction)

4 x 67

```
  6 7
x   4
-----
2 6 8
  2
```

Place the 'carry' digit below the line

7 x 89

```
  8 9
x   7
-----
6 2 3
  6
```

4 x 378

```
  3 7 8
x     4
-------
1 5 1 2
  3 3
```

Stage 4: Long multiplication method (AP23 onwards)

38 x 57

38 × 57 is approximately 40 × 60 = 2400.

```
      3 8
     x 5 7
     2 6 6
      2  5
 +  1 9 0 0
    1  4
   2 1 6 6
   1
```

Remember to use a zero as a place holder in the second row, as in this case you are multiplying the numbers by 5 (worth 50) so the use of the zero ensures that the place value of the digits are correct.

423 x 68

423 × 68 is approximately 400 × 70 = 28000.

```
     4 2 3
     x 6 8
   3 3 8 4
     1  2
+ 2 5 3 8 0
    1  1
  2 8 7 6 4
     1
```

The same method can be used to multiply decimals, such as money.

4.23 x 68

£4.23 × 68 is approximately 4 × 70 = 280.

```
    £4.2 3
     x 6 8
   3 3 8 4
     1  2
+ 2 5 3 8 0
    1  1
 £2 8 7.6 4
     1
```

When multiplying decimals together, count the number of decimals in the question (in the case below 3) and your answer will need to be written to 3 decimal points.

4.23 x 6.8

```
     4.2 3
     x 6.8
   3 3 8 4
     1  2
+ 2 5 3 8 0
    1  1
  2 8.7 6 4
     1
```

When multiplying two decimals, it is important to understand that the position of the decimal point in the answer will not align with the question, as shown in the example opposite.

This can be checked by using the following estimation:

4.23 × 6.8 is approximately 4 × 7 = 28.

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division** which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

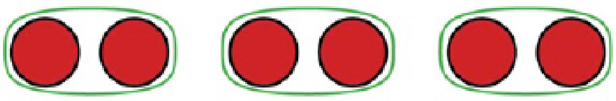
- **understand division as repeated subtraction;**
- **estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;**
- **multiply a two-digit number by a single-digit number mentally;**
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)**
- **subtract numbers using the column method.**

The above points are crucial. If children do not have a secure understanding of these prior learning objectives then they are unlikely to divide with confidence or success, especially when attempting the short and long methods of division.

Stage 1: Practical division, using objects and pictures. (AP10 onwards)

Begin by using practical equipment (cubes or counters) to introduce the '÷' symbol through sharing, e.g. 10 ÷ 2 = 5

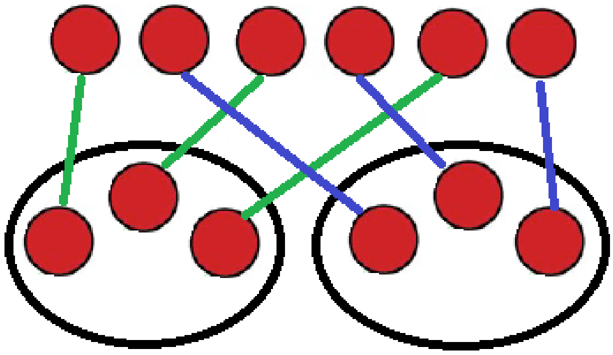
Grouping



How many groups of 2 can I make from 6?

Answer = 3

Sharing



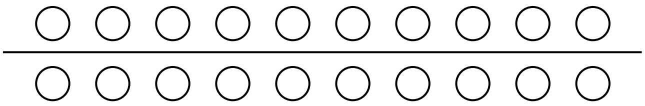
If I share 6 into 2 equal amounts, how many in each group?

Answer = 3

Stage 2: Using arrays (AP11 onwards)

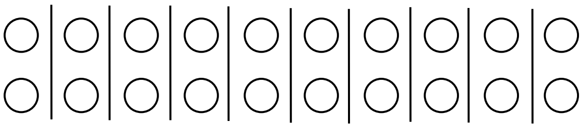
Arrays are a great way to model division, as well as the relationship between multiplication and division.

Using an array



20 ÷ 2 = 10

Or



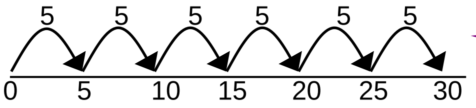
20 ÷ 10 = 2

Stage 3: Division using number lines. (AP12 onwards)

Start to emphasise grouping over sharing as a more efficient way to divide.

Beginning with the number line is an excellent way of linking division to multiplication. It can show division as counting forward to find how many times one number 'goes into' another.

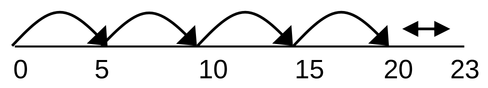
30 ÷ 5



Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.

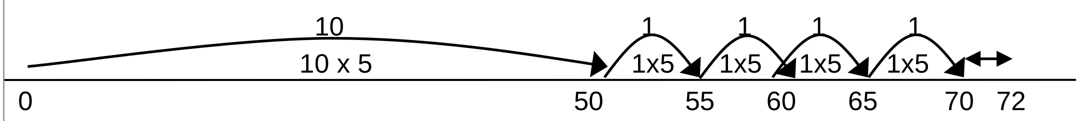
It also helps the children to deal with remainders.

23 ÷ 5 = 4 r 3

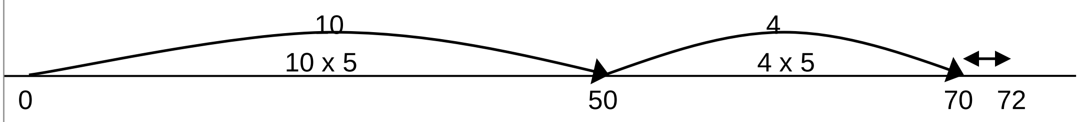


The number line is also an excellent way of introducing the 'chunking' approach.

72 ÷ 5 = 14 r 2



Into a more efficient



Stage 4: Short division using the 'bus stop' method. (AP20 onwards)

Then, refine the method into the traditional format, ensuring that all initial teaching is accompanied by a clear explanation of how this method works (see speech bubbles)

```
   2
3 ) 8 ²7
```

What is 8 ÷ 3? Or how many groups of 3 can you make using 8?

Carry the remaining 2 to the units.

```
   2 9
3 ) 8 ²7
```

What is 27 divided by 3? Or 'How many 3s go into 27?'

When this method is secure for TU ÷ U then quickly progress to HTU ÷ U

Again, begin by briefly linking the method to 'chunking', using numbers where there is no carrying in the hundreds.

222 ÷ 6 = 37 Refine the method, whilst clearly explaining the place value understanding.

```
      3
6 ) 2²2⁴2
```

From 220,what is the largest number of 10s that will divide exactly by 6? 220 ÷ 6 = 30 (or 3 tens). Carry the remaining 40 to the units.

```
      3 7
6 ) 2 2 ⁴2
```

What is 42 divided by 6? Or 'How many 6s go into 42?'

An alternative is to say ***'How many 6s in 220 – the answer must be a multiple of 10'***

Finally, introduce examples of HTU ÷ U where there are also hundreds that need carrying, and where there are remainders. Continue to emphasise the place value until the children are secure with this method.

```
    1
4 ) 5 ¹8  3
```

From 500,what is the largest number of 100s that will divide exactly by 4? 400 ÷ 4 = 100. Carry the remaining 100 to the ten.

Or, 'How many 4s in 500? The answer must be a multiple of 100.

```
    1  4
4 ) 5 ¹8 ²3
```

From 180,what is the largest number of 10s that will divide exactly by 4? 180 ÷ 4 = 40. Carry the remaining 20 to the units.

Or, 'How many 4s in 180? The answer must be a multiple of 10.

```
    1  4  5 R3
4 ) 5 ¹8 ²3
```

What is 23 divided by 4? Or 'How many 4s go into 23?'

Stage 5: Long division using the expanded method. (AP26 onwards)

432 ÷ 15 = 28 r12

```
        2  8 r 12
15 ) 4  3  2
   - 3  0  0       (15 x 20)
     1  3  2
   - 1  2  0       (15 x 8)
        1  2
```

The working in brackets aren't essential, but help develop the children's understanding early on.

In Year 6, children are expected to convert remainders to fractions.

r12 (of 15) $\frac{12}{15} = \frac{4}{5}$

So the answer would be 28 $^4/_5$

Progressing to division of decimals.

432 ÷ 15 = 28.8

```
        2  8 . 8
15 ) 4  3  2 . 0
   - 3  0  ↓
     1  3  2
   - 1  2  0   ↓
        1  2  0
        1  2  0
              0
```